William L. Root; and in Physics, George K. Burgess, William D. Coolidge and Ralph R. Lawrence.

DR. E. LESSER has been appointed associate professor of dermatology at Berlin and Dr. Chermak to the chair of comparative anatomy and embryology at Dorpat. Dr. Winkler, professor of chemistry, has been appointed director of the School of Mines at Freiberg i. S., and Dr. Godschmidt has been promoted to an assistant professorship of chemistry in the University of Heidelberg.

DISCUSSION AND CORRESPONDENCE.

THE STRAIGHT LINE AS A MINIMUM LENGTH.

TO THE EDITOR OF SCIENCE: In looking over the beautiful new text-book of geometry by Profs. Phillips and Fisher one meets with the following proposition of spherical geometry:

The shortest line that can be drawn on the surface of a sphere between two points is the arc of a great circle, not greater than a semi-circumference, joining these points.

The demonstration given is one which has been given before. It appears, for example, in the treatise of Chauvenet (1869) and also in that of George Bruce Halsted (1885). In connection with this demonstration, the reader can hardly escape noticing that every step of it applies equally well to plane geometry. In fact, it is perfectly easy for any student of Euclid's Elements to construct, step by step, a precisely similar proof of the corresponding proposition of plane geometry:

The shortest line that can be drawn between any two points is the straight line which joins them.

The definition of a straight line given by Profs. Phillips and Fisher, therefore, embodies a statement capable of deduction from the geometrical axioms by a chain of logical reasoning, and as a definition, is on strictly scientific grounds, quite indefensible.

Upon examining Prof. Halsted's book, the definitions of which more closely conform to the Euclidean models, one naturally wonders why this demonstration, even more simple in plane than in spherical geometry, has been introduced only in connection with spherical geometry; and one is led to inquire at how early a point the proposition of plane geometry could properly be introduced.

In attempting to establish between any two lines a relation of equality or inequality, we find ourselves compelled to start from the following principles: *The whole is greater than any of its parts; The whole is equal to the sum of all its its parts; Lines which may be placed so as to coincide are equal.* Using these principles alone, it is evident that we cannot compare every two arbitrary lines in magnitude. In any such comparison we must be able to place one of the lines, or portions of it, in complete or partial coincidence with the other. No direct comparison can be instituted, for example, between a straight line and a line no part of which is straight. For the purposes of the proposition in question, therefore, it is necessary to make the distinct assumption, that *the magnitude of every line is comparable with the magnitude of every other line, and between these magnitudes there exists a relation either of equality or of inequality;* or else, what is better, to await the method of limits and the development, by means of it, of metrical ideas, not only for straight lines, but also for curves. Prof. Halsted, accordingly, in spite of his apparent lateness in introducing the proposition, is guilty of an error in theory. He has attempted to give a complete discussion of a proposition, and appears to believe that he has done so, when in reality assumptions additional to those previously made must be introduced before such a discussion can be undertaken.

It seems worth while to make these criticisms, because the two books above referred to are at other points remarkable for their scientific accuracy, and are of so high an order of excellence generally that the student may not readily appreciate the existence of such errors as occur.

THOMAS S. FISKE.

SEPTEMBER 30, 1896.

'A CURVE-TRACING TOP,' AND A CURIOUS OPTICAL ILLUSION.

EDITOR OF SCIENCE: If Prof. Barus will use a smoked glass for his curve-tracing top to spin on, he will get more beautiful tracings than with any lead pencil arrangement. Then let him flow it over with *thin* demar varnish, and